Testing the Standard Model with the electron g-2

Massimo Passera
INFN Padova

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Based on:

- **G.F. Giudice P. Paradisi MP - arXiv:1208.6583**
- **M. Fael MP - arXiv:1402.1575**

The electron g-2: the basics

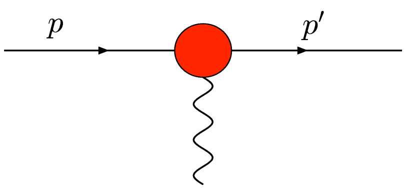
- **The Dirac theory predicts for a lepton l=e,μ,τ**

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- **QFT predicts deviations from the Dirac value:**

$$g_l = 2\,(1 + a_l)$$

- **Study the photon–lepton vertex:**



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \ldots\right] u(p)$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

A pure "quantum correction" effect!

The QED prediction of the electron g-2

$$a_e^{QED} = + \ (1/2)(\alpha/\pi) \ - \ 0.328\,478\,444\,002\,55(33)\ (\alpha/\pi)^2$$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\ldots$$

$$A_2^{(4)}\,(m_e/m_\mu) = 5.197\,386\,68\ (26) \times 10^{-7}$$

$$A_2^{(4)}\,(m_e/m_\tau) = 1.837\,98\ (33) \times 10^{-9}$$

$$+ \ 1.181\,234\,016\,816\ (11)\ (\alpha/\pi)^3$$

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\,241\,456\,587\ldots$$

$$A_2^{(6)}\,(m_e/m_\mu) = -7.373\,941\,62\ (27) \times 10^{-6}$$

$$A_2^{(6)}\,(m_e/m_\tau) = -6.5830\ (11) \times 10^{-8}$$

$$A_3^{(6)}\,(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\ (34) \times 10^{-13}$$

$$- \ 1.9097\ (20)\ (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012;
Kurz, Liu, Marquard & Steinhauser 2014: analytic mass dependent part.

$$+ \ 9.16\ (58)\ (\alpha/\pi)^5$$ **COMPLETED! (12672 mass independent diagrams!)**

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807.

The SM prediction of the electron g-2

- **The SM prediction is:**

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- **The EW (1&2 loop) term is:** Czarnecki, Krause, Marciano '96 [Codata 2012]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- **The Hadronic contribution is:** Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97

$$a_e^{HAD} = 16.82\,(16) \times 10^{-13}$$

- **Which value of α should we use to compute a_e^{SM} and compare it with a_e^{EXP} ?? Not the PDG/Codata one (obtained equating $a_e^{SM}(\alpha) = a_e^{EXP}$)! Use atomic-physics measurements of alpha.**

The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $\boxed{a_e^{SM}(\alpha) = a_e^{EXP}}$ → best determination of alpha (2014):

$$\alpha^{-1} = 137.035\,999\,173\ (34) \qquad [0.25\ \text{ppb}]$$

- Compare it with other determinations (independent of a_e):

$\alpha^{-1} = 137.036\,000\,0\ (11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

$\alpha^{-1} = 137.035\,999\,049\ (90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

The electron g-2: SM vs. Experiment

- **Using α = 1/137.035 999 049 (90) [^{87}Rb, 2011], the SM prediction for the electron g-2 is**

$$a_e^{SM} = 115\,965\,218\,17.8\ \underbrace{(0.6)}_{\delta C_4^{qed}}\ \underbrace{(0.4)}_{\delta C_5^{qed}}\ \underbrace{(0.2)}_{\delta a_e^{had}}\ \underbrace{(7.6)}_{\text{from } \delta\alpha} \times 10^{-13}$$

- **The EXP-SM difference is:**

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -10.5\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1.3σ).
NB: The 4-loop contrib. to a_e^{QED} is $-556 \times 10^{-13} \sim 70\ \delta\Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- **The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):**

$$\underbrace{(0.6)_{\mathrm{QED4}}, \quad (0.4)_{\mathrm{QED5}}, \quad (0.2)_{\mathrm{HAD}},}_{(0.7)_{\mathrm{TH}}} \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\mathrm{EXP}}}$$

⬅ **may drop to 0.2 or 0.3**

- **The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.**

F. Terranova & G.M. Tino, PRA89 (2014) 052118; S. Sturm et al, Nature 506 (2014) 467

→ **sensitivity of 10^{-13} may be reached with ongoing exp. work**

- **In a broad class of BSM theories, contributions to a_l scale as**

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\boxed{\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) \; 0.7 \times 10^{-13}}$$

The electron g-2 sensitivity and NP tests (II)

- The experimental sensitivity in Δa_e is not far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

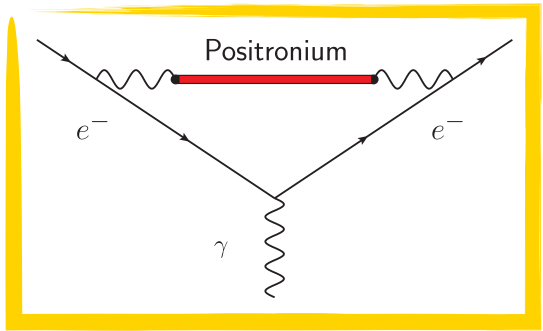
- BSM scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e (& Δa_τ) and contributions to EDMs, LFV or lepton universality breaking observables.

- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

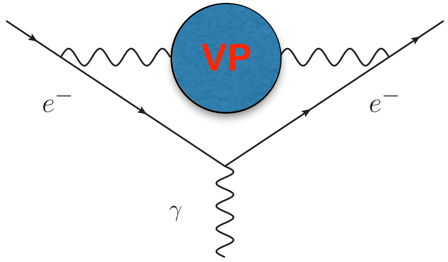
Is there a positronium contribution to the electron g-2?

- **The leading contribution of positronium to a_e comes from:**

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690; Eides 1402.5860; Hayakawa 1403.0416



- **The e^+e^- bound states appear as poles in the vac. pol. $\Pi(q^2)$ right below the branch-point $q^2 = (2m)^2$. Their contribution is:**



$$a_e(\mathrm{vp}) = \frac{\alpha}{\pi^2} \int_0^\infty \frac{ds}{s} \, \mathrm{Im}\,\Pi(s + i\epsilon)\, K(s)$$

$$a_e^{\mathrm{P}} = \frac{\alpha^5}{4\pi} \zeta(3) \left(8 \ln 2 - \frac{11}{2} \right) = 0.9 \times 10^{-13} = 1.3 \left(\frac{\alpha}{\pi} \right)^5$$

Mishima 1311.7109; Fael & MP 1402.1575

Is there a positronium contribution to the electron g-2? (II)

- This result is of the same magnitude of the experimental uncertainty of a_e and of the same order of α as the 5-loop one!

- Melnikov et al 1402.5690 determined a nonpert. contrib. of the e^+e^- continuum right above threshold that cancels one-half of a_e^P:

$$a_e(\mathrm{vp})^{\mathrm{cont,np}} = -\frac{|\alpha|^5}{8\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right)$$

- In fact the total positronium poles + continuum nonperturbative contribution to a_e arising from the threshold region at LO in α is:

$$a_e^{\mathrm{thr}}(\mathrm{vp}) = -\frac{\alpha}{\pi}K(4m^2)\,\mathrm{Re}\,A(1)$$

with

$$A(\beta) = -\frac{\alpha^2}{2}\left[\gamma + \psi\left(1 - \frac{i\alpha}{2\beta}\right)\right] = \frac{\alpha^2}{2}\sum_{k=1}^{\infty}\zeta(k+1)\left(\frac{i\alpha}{2\beta}\right)^k$$

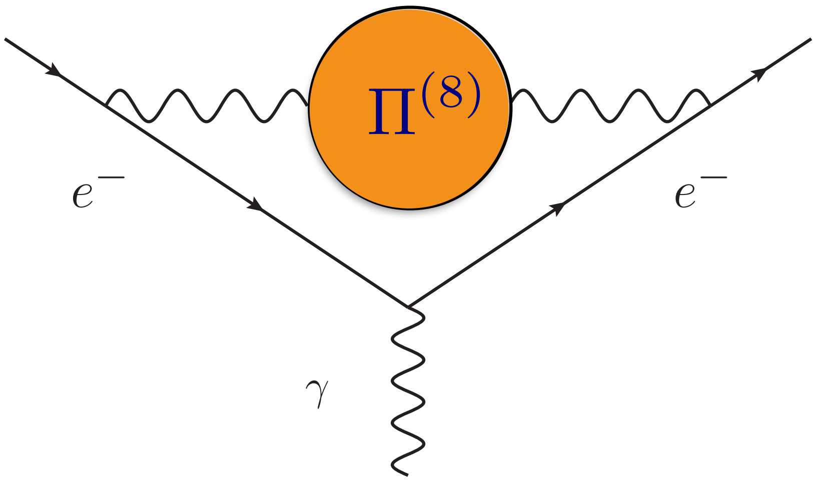
so that

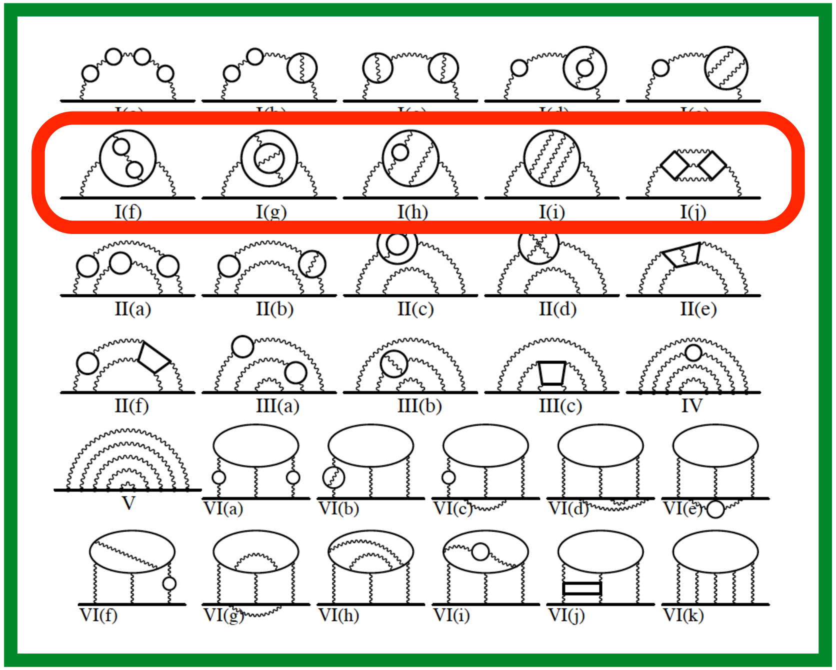
$$\boxed{a_e^{\mathrm{thr}}(\mathrm{vp}) = \frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) = \frac{a_e^{\mathrm{P}}}{2}}$$

Is there a positronium contribution to the electron g-2? (III)

- So, should we add this total threshold contribution $a_e^P/2$ to the perturbative QED 5-loop result of Kinoshita and collaborators?

- The 5-loop QED contribution to a_e arising from the insertion of the 4-loop VP in the photon line has been computed via



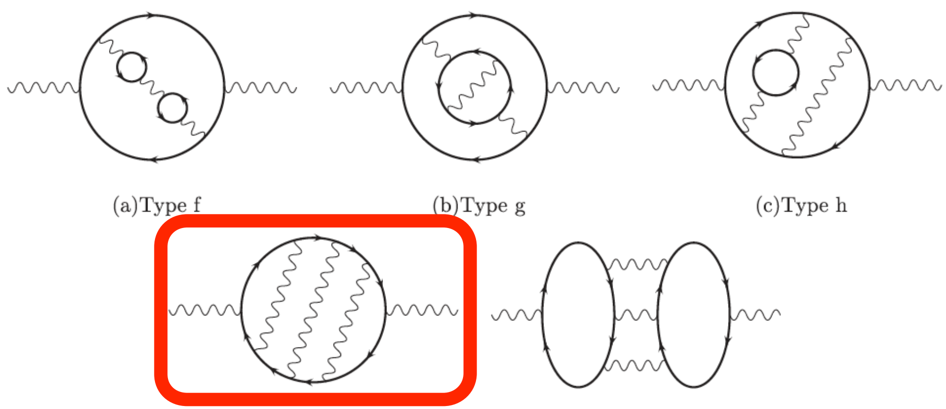


Aoyama, Hayakawa, Kinoshita & Nio 2012

- **Using explicit expressions for $\Pi^{(8)}(q^2)$ (Baikov, Maier, Marquard '13) we obtain:**

$$a_e^{(10)}(\mathrm{vp}) = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Pi^{(8)}\!\left(-\frac{m^2x^2}{1-x}\right)$$

$$\boxed{a_e^{(10)}(\mathrm{vp}) = \frac{a_e^{\mathrm{P}}}{2} + \cdots}$$



(a)Type f (b)Type g (c)Type h

(d)Type i (e)Type j

- **$a_e^P/2$ is already included in the 5-loop contrib → don't add it!**
- **There is no additional contrib of QED bound states beyond PT**

(M.A. Braun 1968; Barbieri, Christillin, Remiddi 1973; Melnikov et al 1402.5690; Eides 1402.5860)

Conclusions

- The uncertainty of the SM prediction of a_e is dominated by the exp uncertainty of alpha. A robust and ambitious exp program is under way to improve both α & a_e.
- It may soon be possible to test NP with a_e. In particular, whether the muon g-2 discrepancy manifests itself also in the electron g-2!
- The positronium contribution to a_e should not be added to that of perturbative QED. There is no additional contrib. of QED bound states beyond perturbation theory.

The End